Ratio proportions and mixtures:

Analogy of a mixture containing A and B in the ratio p:q after 'n' replacements of quantity 'y' from mixture with the same quantity of pure B from a total quantity of 'x':

Let 'x' be the initial quantity of a liquid mixture containing components A and B in the ratio p:q.

First replacement of quantity 'y' with equal quantity of pure B:

If some 'y' quantity of the mixture is replaced with pure B,

Quantity of A becomes $(p/(p+q))*(x-y)$

Ratio of A with that of the mixture is, $(p/(p+q))*((x-y)/x)$

Second replacement of quantity 'y' with equal quantity of pure B:

If again, some 'y' quantity of the mixture is replaced with pure B,

Quantity of A becomes $(p/(p+q))*((x-y)^2/x)$

Ratio of A with that of the mixture is, $(p/(p+q))*((x-y)^2/x^2)$

Hence,

After 'n' replacements of quantity 'y' with equal quantity of pure B:

Quantity of A in the mixture = $(p/(p+q))*((x-y)^n/x^{n-1})$----------(1)

Ratio of A with that of the mixture = $(p/p+q)* ((x-y)^n/x^n)$----------(2)

Ratio of B with that of the mixture = $1- ((p/(p+q)*((x-y)^n/x^n))$

$= (((p+q)*x^n)-(p*(x-y)^n))/((p+q)*x^n)$------(3)

Quantity of B in the mixture = $(((p+q)*x^n)-(p*(x-y)^n))/((p+q)*x^{n-1})$----------(4)

Ratio of A and B = $(p*(x-y)^n)/(((p+q)*x^n)- (p*(x-y)^n))$----------(5)

Example1 : If 5 litres from a 25 litres mixture containing milk and water in the ratio 3:2 is replaced thrice with equal quantity of water in each occasion,

1. What is the quantity of milk in the mixture after the process?
2. What is the ratio of milk to that of milk and water mixture after the process?
3. What is the quantity of water in the mixture after the process?
4. What is the ratio of water to that of milk and water mixture after the process?
5. What is the ratio of milk and water after the process?

Solution:

1. Quantity of milk = $(3/5)*(20^3/25^2)$ (From equation (1)).
   = (192/25) litres.
2. Ratio of milk to that of the mixture = $(3/5)*(20^3/25^3)$ (From equation (2))

= (192/625).

3. Quantity of water in the mixture = $((5*25^3)-(3*20^3))/(5*25^2)$ (From equation (4))
   = (433/25) litres.
4. Ratio of water to that of the mixture = $((5*25^3)-(3*20^3))/(5*25^3)$ (From equation (3))
   = (433/625).
5. Ratio of milk and water = $(3*20^3)/((5*25^3)-(3*20^3))$----------(From equation (5))
   = (192/433).